

THE MUON PARADOX: EVIDENCE OF SPECIAL RELATIVITY IN NATURE

SIDDHARTH SELVARAJ

Abstract

Muons are unstable elementary particles produced when high-energy cosmic rays collide with atoms in Earth's atmosphere. Although muons have a proper lifetime of only 2.2 microseconds, large numbers are detected at Earth's surface. This contradiction presents what is known as the "muon lifetime problem." The purpose of this project was to investigate how muons are able to travel 10–15 kilometers through Earth's atmosphere before decaying. Research shows that muons are formed when particle showers, like cosmic rays, collide and generate pions, which rapidly decay into muons. Using classical physics, a muon traveling near the speed of light should only travel about 660 meters before decaying, calculated using $d=vt$. This is much less than the thickness of the atmosphere. However, according to Einstein's theory of special relativity, time dilation occurs for objects moving at velocities close to the speed of light. The relativistic time equation is $t=\gamma\tau$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. The proper lifetime of a muon ($\tau_0 = 2.2 \mu\text{s}$) is extended in Earth's frame according to the equation, as it predicts that a muon traveling at 0.998c experiences a lifetime extended by a gamma factor of about 15. This allows many muons to survive long enough to reach Earth's surface. The detection of atmospheric muons provides direct, measurable evidence of special relativity in nature.

Inspiration and Focus

The Spark: I have always been captivated by phenomena that challenge our perception of time and space and the mysteries of the infinitesimal world. The Muon Paradox is a perfect example of a world of relativistic effects happening right above our heads at the subatomic level.

The Goal: I wanted to move beyond the "what" and understand the mathematical "how" that bridges the gap between a 2.2-microsecond lifetime and a 15-kilometer journey.

Narrowing the Scope: Rather than exploring all cosmic rays, I focused specifically on the muon's frame of reference to see how Einstein's equations resolve the conflict between classical and relativistic physics.

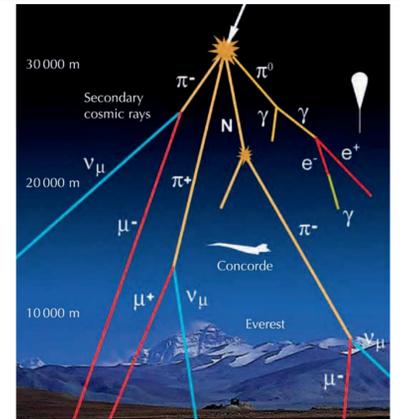


Fig: Cosmic Rays [9]

What is a Muon?

LEPTONS		
0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²
-1	-1	-1
1/2	1/2	1/2
e	μ	τ
electron	muon	tau
<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²
0	1/2	1/2
ν_e	ν_μ	ν_τ
electron neutrino	muon neutrino	tau neutrino

Fig: Three generations of Leptons. [8]

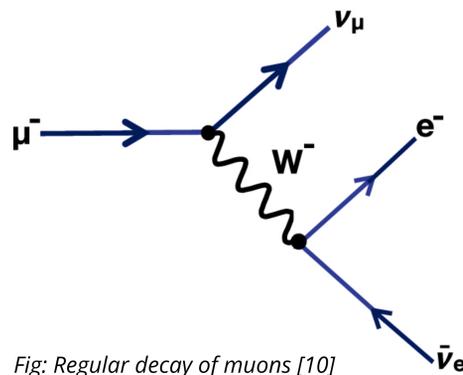


Fig: Regular decay of muons [10]

The "Heavy Electron": Muons are elementary leptons, identical to electrons in charge but roughly 207 times more massive.[1]

Unstable Nature: They are *temporary* particles in our universe, rapidly decaying into an electron and a neutrino pair.[7]

The Birthplace: They are created 10–15 km above us when high-energy cosmic rays collide with atmospheric nuclei, triggering a "particle shower".[1]

The Muon Lifetime Problem (Classical Paradox)

Using Newton's Classical predictions causes inconsistencies with physical observations.

- **Proper Lifetime (τ_0):** 2.2 μs
- **Velocity (v):** $\approx 0.998c$ (very close to the speed of light)

Classical Calculation ($d=vt$):

$$(3 \times 10^8 \text{ m/s}) \times (2.2 \times 10^{-6} \text{ s}) \approx 660 \text{ meters}[1]$$

The Conflict: If muons only travel 660 m, they should all decay in the upper atmosphere. Yet, 10,000 muons strike every square meter of Earth every minute.[5]

THE ROSSI-HALL EXPERIMENT: CONFIRMING TIME DILATION

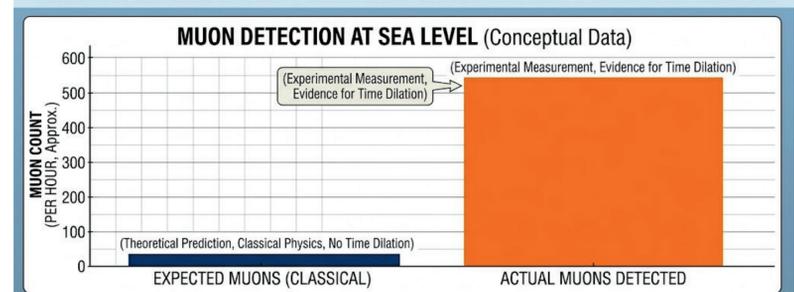


Fig: Bar graph representing results [10]

Muon's Impact on Earth's Surface

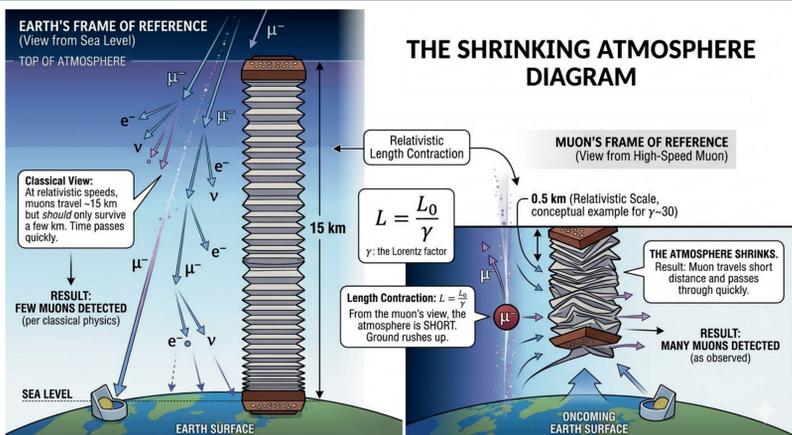


Fig: Length contraction visualization [10]

The Constant Rain: At any given moment, muons are passing through your body. Due to the continuous flux of particles, approximately 1 muon passes through your hand every second.

Energy and Matter: These muons are "ionizing radiation." They have enough kinetic energy to penetrate meters of rock or lead, which is why they reach the "land" and don't just stop at the roof of a building.

Detection: We don't feel them because they interact very weakly with matter, but without Time Dilation/Length Contraction, muons would not reach the surface of the Earth.

Sources/References

- [1] Coan, T.E. & Ye, J., "Muon Physics", Southern Methodist University, September 26, 2003.
- [2] Field, J.H., "Muon decays in the Earth's atmosphere, time dilatation and relativity of simultaneity", arXiv, January 22, 2009, <https://arxiv.org/abs/physics/0606188>
- [3] Lobo, I.P. & Pfeifer, C., "Muon accelerators - Muon lifetime measurements as window to Planck scale physics", arXiv, November 30, 2023, <https://arxiv.org/abs/2306.07210>
- [4] McKelvie, J., "Revisiting the Matter of Time Dilation in Special Relativity", Journal of Physics & Optics Sciences, 2024.
- [5] MIT Department of Physics, "The Speed and Decay of Cosmic-Ray Muons: Experiments in the Relativistic Kinematics of the Universal Speed Limit and Time Dilation", Massachusetts Institute of Technology, September 19, 2024.
- [6] Premović, P.I., "Relativistic Time Dilation and the Muon Experiment", The General Science Journal, April 4, 2020, <https://hal.science/hal-02531926v1>

THE MUON PARADOX: EVIDENCE OF SPECIAL RELATIVITY IN NATURE

SIDDHARTH SELVARAJ

Relativistic Mechanics

Special Relativity is built on two postulates: the laws of physics are the same for everyone, and the speed of light (c) is constant for all observers. Because c cannot change, space and time must change to compensate for high-speed travel.

Perspective A: The Earth Observer (Time Dilation)

The Concept: When we watch a muon travel at $0.998c$, we are observing a "moving clock." According to Einstein, moving clocks run slow relative to a stationary observer.[4]

The "Why": For the muon's internal clock to tick, a subatomic process must occur. From our perspective on Earth, that process is stretched across kilometers of distance. Because the speed of light (c) is the limit, the "tick" takes longer to complete from our point of view.

The Formula: We calculate the Lorentz Factor (γ), which represents the scale of relativistic change:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

At $0.998c$, $\gamma \approx 15.8$.

The Result: We multiply the muon's "rest" lifetime ($\tau_0 = 2.2\mu s$) by γ :

$$t = \gamma \times \tau_0 \rightarrow 15.8 \times 2.2\mu s \approx 35\mu s[6]$$

Scientific Conclusion: Time dilation extends the observed lifetime for the muon from our perspective, giving it enough time to reach the ground before it decays.

THE LIGHT CLOCK: A CLASSIC TIME DILATION DIAGRAM

EXPLORING THE FUNDAMENTALS OF RELATIVITY

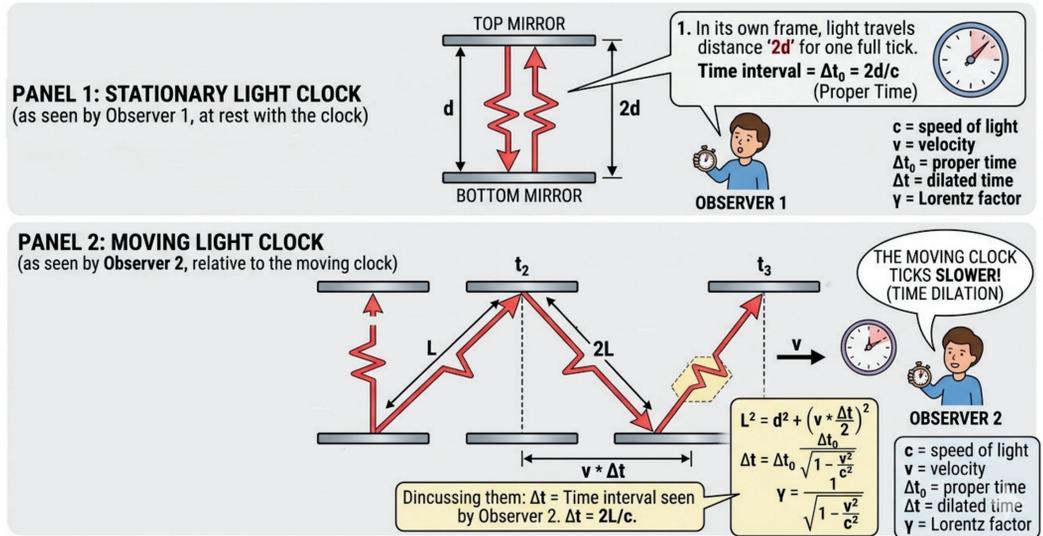


Fig: Moving light clock framework [10]

Length Contraction

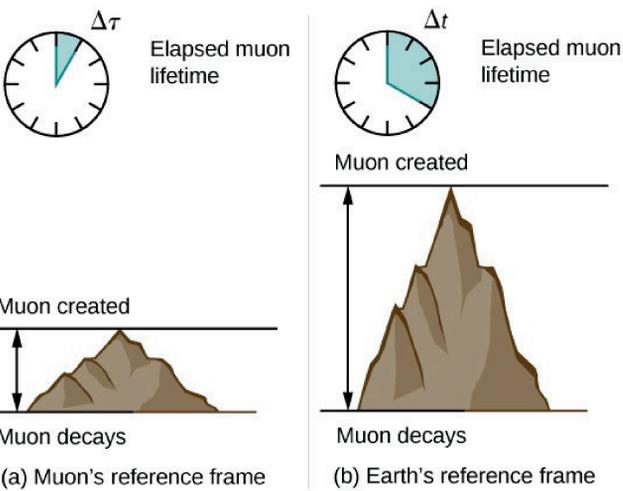


Fig: Length contraction visual [10]

Perspective B: The Muon's Frame (Length Contraction)

The Concept: In the muon's own frame of reference, it is at rest. It doesn't feel like its clock is slow; it feels perfectly normal. However, it sees the Earth rushing toward it at $0.998c$.

The "Why": To keep the speed of light constant, relativity dictates that the distance between two moving points must contract.[2] If the Earth is moving toward the muon, the entire atmosphere (the "ruler" measuring the distance) physically shrinks in the direction of motion.

The Formula: We divide the "proper length" ($L_0 = 15$ km) by the Lorentz Factor (γ):

$$L = L_0 / \gamma \rightarrow 15 \text{ km} / 15.8 \approx 0.95 \text{ km} (950 \text{ meters})^{[2]}$$

The Result: The muon only has to survive a journey of 950 meters, which it can easily do in its standard $2.2 \mu s$ lifetime.

Scientific Conclusion: Time dilation and length contraction are not separate effects, but complementary descriptions of the same relativistic phenomenon observed from different frames of the "Muon Paradox."

Experimental Evidence & Takeaways

Altitude Flux: Experiments prove that muon counts on mountain tops (like Mt. Washington) compared to sea level match Einstein's predictions, not classical ones.[5]

Key Takeaway 1: Classical physics fails at near-light speeds.

Key Takeaway 2: Muons are "living clocks" that prove time is relative, not absolute.

Future Investigations

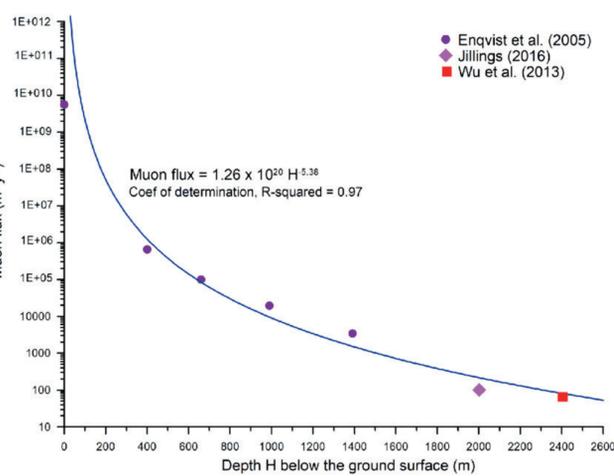


Fig: Muon flux vs Depth [12]

Muon Tomography: How can we use "muon X-rays" to see inside the Great Pyramids or volcanoes?

National Security: Can muon detectors identify shielded nuclear materials in shipping containers?

Deep Space: What do muon production rates tell us about the massive supernovae that create cosmic rays?

MUON TOMOGRAPHY: REVEALING THE INTERNAL STRUCTURE OF A VOLCANO

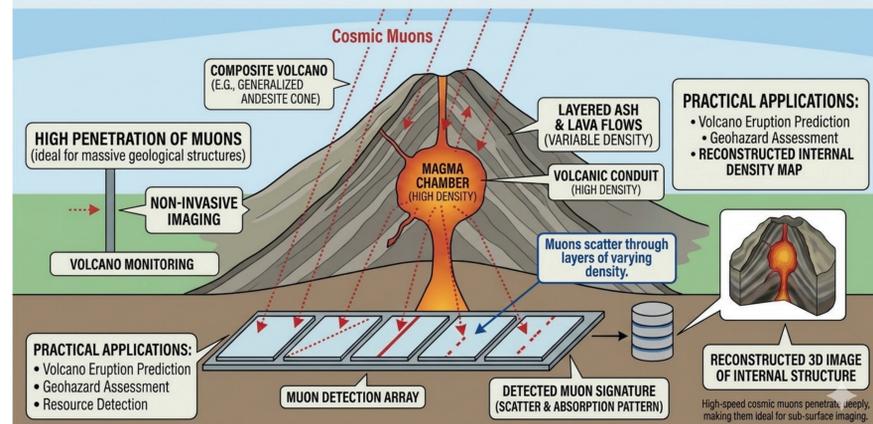


Fig: Volcano Topography [10]

Sources/References

[7] Sleator, T. & Budick, B., "The Muon Lifetime", New York University, April 5, 2012.
 [8] Siegel, Ethan. "Ask Ethan: Why Are There Only Three Generations of Particles?" Forbes, 21 Sept. 2019, www.forbes.com/sites/startswithabang/2019/09/21/ask-ethan-why-are-there-only-three-generations-of-particles/.
 [9] Lodovico. "Cosmic Muons Decay." PhysicsOpenLab, 10 Jan. 2016, physicsopenlab.org/2016/01/10/cosmic-muons-decay/.
 [10] Images generated by Google's Gemini based on a prompt provided to visually explain the theory
 [11] Exchange forces in Particle Physics - https://link.springer.com/article/10.1007/s10701-021-00425-0
 [12] Zhang, Zongxian, et al. "Muography and Its Potential Applications to Mining and Rock Engineering." Journal of Rock Mechanics and Geotechnical Engineering, vol. 12, no. 6, Nov. 2020, pp. 1-22. ResearchGate